

Solve:  $\overset{6x}{\left(\frac{1}{x}\right)} + \overset{3}{\left(\frac{1}{3}\right)} = \overset{6x}{\left(\frac{5}{6}\right)}$

$$\frac{6x}{x} + \frac{6x}{3} = \frac{30x}{6}$$

$$6 + 2x = 5x$$

$$6 = 3x$$

$$\boxed{x=2}$$

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$x \neq 0$$

$$x = x$$

$$3 = 3$$

$$6 = 3 \cdot 2$$

$$\text{LCD} = 3 \cdot 2 \cdot x \\ = 6x$$

Solve:  $\left(\frac{1}{y}\right)^{15y} + \left(\frac{2}{3}\right)^{15y} = \left(\frac{1}{5}\right)^{15y}$

$$15 + 10y = 3y$$

$$15 = -7y$$

$$y = -\frac{15}{7}$$

$$y \neq 0$$

$$y = y \cdot 1$$

$$3 = 3$$

$$\frac{5 = 5}{LCD = 15y}$$

$$y \cdot 3 \cdot 5$$

$$\frac{1}{-\frac{15}{7}} + \frac{2}{3} = \frac{1}{5}$$

$$-\frac{7}{15} + \frac{10}{15} = \frac{1}{5}$$

$$\frac{3}{15} = \frac{1}{5}$$

## HOW TO

### **Solve equations with rational expressions.**

- Step 1. Note any value of the variable that would make any denominator zero.
- Step 2. Find the least common denominator of *all* denominators in the equation.
- Step 3. Clear the fractions by multiplying both sides of the equation by the LCD.
- Step 4. Solve the resulting equation.
- Step 5. Check:
- If any values found in Step 1 are algebraic solutions, discard them.
  - Check any remaining solutions in the original equation.

Solve:  $(1) - \left(\frac{5}{y}\right) = -\left(\frac{6}{y^2}\right)$

$$y^2 - 5y = -6$$

$$y^2 - 5y + 6 = 0$$

$$(y-3)(y-2) = 0$$

$$y-3=0 \quad y-2=0$$

$$y=3 \quad y=2$$

$$y=0$$

$$\text{LCD} = y^2$$

$$1 - \frac{5}{y} = -\frac{6}{y^2}$$

$$\frac{y}{y} - \frac{5}{y} = -\frac{6}{y^2}$$
$$-\frac{4}{y} = -\frac{6}{y^2}$$

$$1 - \frac{4y}{y} = -\frac{6}{y}$$

$$\frac{y}{y} - \frac{4y}{y} = -\frac{6}{y}$$
$$-\frac{3}{y} = -\frac{6}{y}$$

Solve:  $\left(\frac{1}{x}\right)^{x^2} - \left(\frac{2}{x}\right)^{x^2} = \left(\frac{15}{x^2}\right)^{x^2}$

$$x \neq 0$$

$$x^2 - 2x = 15$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \quad x = -3$$

$$\text{Solve: } \frac{2}{x+2} + \frac{4}{x-2} = \frac{x-1}{x^2-4}$$

$$x \neq \pm 2$$

$$\text{LCD} = (x+2)(x-2)$$

$$2(x-2) + 4(x+2) = x-1$$

$$2x-4+4x+8 = x-1$$

$$6x+4 = x-1$$

$$5x+4 = -1$$

$$5x = -5$$

$$x = -1$$

$$\frac{2}{-1+2} + \frac{4}{-1-2} = \frac{-1-1}{(-1)^2-4}$$

$$\frac{2}{1} + \frac{4}{-3} = \frac{-2}{-3}$$

$$\frac{6}{3} - \frac{4}{3} = \frac{2}{3}$$

Solve:  $\frac{5}{y+3} + \frac{2}{y-3} = \frac{5}{y^2-9}$

$$y \neq \pm 3$$

$$\text{LCD} = (y+3)(y-3)$$

$$5(y-3) + 2(y+3) = 5$$

$$5y - 15 + 2y + 6 = 5$$

$$7y - 9 = 5$$

$$7y = 14$$

$$y = 2$$

Solve:  $\frac{m+11}{m^2-5m+4} = \frac{5}{m-4} - \frac{3}{m-1}$ .

$$m \neq 4, 1$$

$$\text{LCD} = (m-4)(m-1)$$

$$m+11 = 5(m-1) - 3(m-4)$$

$$m+11 = 5m-5-3m+12$$

$$m+11 = 2m+7$$

$$11 = m+7$$

$$\textcircled{\cancel{m=4}}$$

→ Extraneous Solution

No Solution



Solve:  $\frac{x+13}{x^2-7x+10} = \frac{6}{x-5} - \frac{4}{x-2}$ .

Solve:  $\frac{y}{y+6} = \frac{72}{y^2-36} + 4.$

$$y \neq \pm 6$$

$$\text{LCD} = (y-6)(y+6)$$

$$y(y-6) = 72 + 4(y^2-36)$$

$$y^2 - 6y = 72 + 4y^2 - 144$$

$$y^2 - 6y = 4y^2 - 72$$

$$0 = 3y^2 + 6y - 72$$

$$0 = y^2 + 2y - 24$$

$$0 = (y+6)(y-4)$$

$$\cancel{y = -6} \quad y = 4$$

$$\frac{y}{y+6} = \frac{72}{y^2-36} + 4$$

$$\frac{y}{10} = -\frac{72}{20} + 4$$

$$\frac{y}{10} = -\frac{36}{10} + \frac{40}{10}$$

$$\frac{y}{10} = \frac{4}{10}$$

Solve:  $\frac{x}{x+4} = \frac{32}{x^2-16} + 5.$

$$\left(\frac{x}{2x-2}\right)^{\frac{12(x+1)(x-1)}{2(x-1)}} - \left(\frac{2}{3x+3}\right)^{\frac{12(x+1)(x-1)}{3(x+1)}} = \left(\frac{5x^2-2x+9}{12x^2-12}\right)^{\frac{12(x+1)(x-1)}{12(x+1)(x-1)}}$$

$$x \neq \pm 1$$

$$LCD = 12(x+1)(x-1)$$

$$2x-2 = 2(x-1)$$

$$3x+3 = 3(x+1)$$

$$12x^2-12 = 2(x-1) \cdot 3(x+1) \cdot 2$$

$$\begin{aligned} &12(x^2-1) \\ &12(x-1)(x+1) \\ &2 \cdot 2 \cdot 3(x-1)(x+1) \end{aligned}$$

$$6(x+1) \cdot x - 4(x-1) \cdot 2 = 5x^2 - 2x + 9$$

$$6x^2 + 6x - 8x + 8 = 5x^2 - 2x + 9$$

$$6x^2 - 2x + 8 = 5x^2 - 2x + 9$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x = \pm 1$$

No Solution

$$\frac{y}{5y-10} - \frac{5}{3y+6} = \frac{2y^2-19y+54}{15y^2-60}$$

$$\frac{z}{2z+8} - \frac{3}{4z-8} = \frac{3z^2-16z-16}{8z^2+16z-64}.$$

$$\frac{4}{3x^2-10x+3} + \frac{3}{3x^2+2x-1} = \frac{2}{x^2-2x-3}.$$

Solve:  $\frac{15}{x^2+x-6} - \frac{3}{x-2} = \frac{2}{x+3}$ .



Solve:  $\frac{5}{x^2+2x-3} - \frac{3}{x^2+x-2} = \frac{1}{x^2+5x+6}$ .

$x \neq -4, 2, -5$

$$\frac{2}{x^2+2x-8} - \frac{1}{x^2+9x+20} = \frac{4}{x^2+3x-10}$$

$(x+4)(x-2)$        $(x+5)(x+4)$        $(x+5)(x-2)$

LCD  $(x+4)(x-2)(x+5)$

$$2(x+5) - (x-2) = 4(x+4)$$

$$2x+10 - x+2 = 4x+16$$

$$x+12 = 4x+16$$

$$12 = 3x+16$$

$$-4 = 3x$$
$$x = -\frac{4}{3}$$